Atlanta Lecture Series
in Graph Theory and Combinatorics XXVIII

Georgia Institute of Technology

March 2 - 3, 2024

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Introduction

About

The Atlanta Lecture Series (ALS) in Combinatorics and Graph Theory is a major event for the combinatorics community in the southeast region of the United States, providing opportunities to strengthen collaborations among researchers and institutions within the region and beyond.

ALS has been alternately hosted by Emory University, Georgia Institute of Technology, and Georgia State University, three major research universities in the Atlanta metropolitan area, and is currently hosted by Georgia Institute of Technology and Georgia State University. This conference series is supported by the National Science Foundation, and ALS 28 is also supported by the Georgia Tech School of Mathematics and the Georgia Tech ACO (Algorithms, Combinatorics, and Optimization) Program. It features the latest research developments and themes in the areas of structural graph theory, extremal graph theory, random graphs, hypergraphs, and more. Each conference features one principle speaker and several other outstanding combinatorialists/graph theorists, as well as some promising young researchers.

Local Information

Venue:

Room 005,
Skiles Classroom Building,
686 Cherry St NW, Atlanta, GA 30332

Contacts:

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Conference Schedule

Saturday Afternoon (Skiles 005)		
Time		
11:00 – 11:50	sandwiches	
12:00 – 12:50	Luke Postle	Refined Absorption: A New Proof of the Existence Conjecture
13:00 – 13:50	Michelle Delcourt	Hypergraph Matchings Avoiding Forbidden Submatching
14:00 – 14:20	Evelyne Smith-Roberge	Correspondence Packings of Planar Graphs
14:30 – 14:50	coffee	
15:00 – 15:50	Jie Ma	Supersaturation Beyond Color-Critical Graphs
16:00 – 16:50	Matija Bucic	Robust Sublinear Expanders
17:00 – 17:20	Zihan Tan	Near-Linear Approximate Emulators for Planar Graphs
18:30	Banquet at the GT Hotel (second floor, room A)	

Sunday Morning (Skiles 005)		
Time		
8:00 – 8:50	Eric Ramos	The Graph Minor Theorem in Topology
9:00 – 9:20	Zhiyu Wang	Oriented Diameter of Some Graph Classes
9:30 – 9:50	Ruth Luo	The Existence of a Spanning Jellyfish
10:00 – 10:20	coffee	
10:30 – 11:20	Luke Postle	Refined Absorption: Further Applications
11:30 – 11:50	Jagdeep Singh	Apexing and Edge-Apexing in Graphs
12:00 – 12:50	Tibor Szabó	New Ramsey Multiplicity Bounds and Search Heuristics
13:00	sandwiches	

Abstracts

Refined Absorption: A New Proof of the Existence Conjecture

Luke Postle

University of Waterloo

The study of combinatorial designs has a rich history spanning nearly two centuries. In a recent breakthrough, the notorious Existence Conjecture for Combinatorial Designs dating back to the 1800s was proved in full by Keevash via the method of randomized algebraic constructions. Subsequently Glock, Kühn, Lo, and Osthus provided an alternate purely combinatorial proof of the Existence Conjecture via the method of iterative absorption. We introduce a novel method of "refined absorption" for designs; in this talk, as our first application of the method we provide a new alternate proof of the Existence Conjecture. Joint work with Michelle Delcourt.

Hypergraph Matchings Avoiding Forbidden Submatchings

Michelle Delcourt

Toronto Metropolitan University

In 1973, Erdős conjectured the existence of high girth $(n, 3, 2)$-Steiner systems. Recently, Glock, Kühn, Lo, and Osthus and independently Bohman and Warnke proved the approximate version of Erdős' conjecture. Just this year, Kwan, Sah, Sawhney, and Simkin proved Erdős' conjecture. As for Steiner systems with more general parameters, Glock, Kühn, Lo, and Osthus conjectured the existence of high girth (n, q, r)-Steiner systems. We prove the approximate version of their conjecture. This result follows from our general main results which concern finding perfect or almost perfect matchings in a hypergraph G avoiding a given set of submatchings (which we view as a hypergraph H where $V(H) = E(G)$). Our first main result is a common generalization of the classical theorems of Pippenger (for finding an almost perfect matching) and Ajtai, Komlós, Pintz, Spencer, and Szemerédi (for finding an independent set in girth five

hypergraphs). More generally, we prove this for coloring and even list coloring, and also generalize this further to when H is a hypergraph with small codegrees (for which high girth designs is a specific instance). Indeed, the coloring version of our result even yields an almost partition of K_n^r into approximate high girth (n, q, r)-Steiner systems. If time permits, I will explain some of the other applications of our main results such as to rainbow matchings. This is joint work with Luke Postle.

Correspondence Packings of Planar Graphs

Evelyne Smith-Roberge

Georgia Institute of Technology

Suppose a graph G has list chromatic number k. It is easy to see that if L is a $(k+1)$-list assignment for G, then G admits two L-colourings φ_1 and φ_2 where $\varphi_1(v) \neq \varphi_2(v)$ for every $v \in V(G)$. But what if we want still more disjoint L-colourings without making our lists too big? In this talk, I will discuss recent progress towards determining the list packing number of various classes of planar graphs: that is, the smallest number k such that if L is a k-list assignment for an arbitrary graph G in the class under study, then L can be decomposed into k disjoint L-colourings. All results I will discuss also hold in the correspondence colouring framework. Joint work with Daniel Cranston.

Supersaturation Beyond Color-Critical Graphs

Jie Ma

University of Science and Technology of China

A classic result of Rademacher from 1941 led to the study of supersaturation problems of graphs, which aim to count the minimum number of copies of a given graph F among all graphs with n vertices and m edges. This is closely related to a central concept in Extremal Graph Theory – the Turán number of F. Famous results of Erdős, and Lovász and Simonovits determine the minimum number of cliques K_r in graphs whose number of edges exceed the Turán number of K_r by a linear term $O(n)$. Subsequent works of Mubayi as well as Pikhurko and Yilma extend these classical results from cliques to color-critical graphs, a rich family playing important roles in extremal problems. In this talk, we will discuss supersaturation problems beyond color-critical graphs and investigate natural enumerative parameters. Our results go beyond the previous results and show that supersaturation problems for general graphs can be rather complicated. Among others, we disprove a conjecture of Mubayi. Joint work with Long-Tu Yuan.

Robust Sublinear Expanders

Matija Bucic

Princeton University

Expander graphs are perhaps one of the most widely useful classes of graphs ever considered. In this talk, we will focus on a fairly weak notion of expanders called sublinear expanders, first introduced by Komlós and Szemerédi around 30 years ago. They have found many remarkable applications ever since. In particular, we will focus on certain robustness conditions one may impose on sublinear expanders and some applications of this idea, which include:

– recent progress on the classical Erdős-Gallai cycle decomposition conjecture,

– essentially tight answer to the classical Erdős unit distance problem for "most" real normed spaces and

– an asymptotic solution to the rainbow Turan problem for cycles, raised by Keevash, Mubayi, Sudakov and Verstraete, with an interesting corollary in additive number theory.

Near-Linear Approximate Emulators for Planar Graphs

Zihan Tan

Rutgers University

We study vertex sparsification for distances, in the setting of planar graphs with distortion: Given a planar graph G (with edge weights) and a subset of k terminal vertices, the goal is to construct an ϵ-emulator, which is a small planar graph G' that contains the terminals and preserves the distances between the terminals up to factor $1+\varepsilon$. We construct the first ε-emulators for planar graphs of near-linear size $\tilde{O}(k/poly(\varepsilon))$. In terms of k, this is an improvement over the previous quadratic upper bound of Cheung, Goranci, and Henzinger, and breaks below known quadratic lower bounds for exact emulators (the case when $\varepsilon = 0$). Moreover, our emulators can be computed in (near-)linear time, which leads to fast $(1+\varepsilon)$- approximation algorithms for basic optimization problems on planar graphs, including multiple-source shortest paths, minimum (s,t)-cut, graph diameter, and offline dynamic distance oracle. This talk is based on joint work with Hsien-Chih Chang and Robert Krauthgamer.

The Graph Minor Theorem in Topology

Eric Ramos

Stevens Institute of Technology

The Graph Minor Theorem of Robertson and Seymour is one of the most celebrated results in the history of combinatorics, spanning decades (and hundreds of pages) of work. In this talk we discuss recent work of Miyata, Proudfoot, and the author that proposes a framework that would allow one to apply the Graph Minor Theorem to algebra and topology, building on seminal contributions by Sam and Snowden. Specifically, we will discuss applications of this framework to the study of matching complexes and configuration spaces of graphs.

Oriented Diameter of some Graph Classes

Zhiyu Wang

Louisiana State University

The oriented diameter of an undirected graph G is the minimum diameter of an orientation of G over all strongly connected orientations of G. In this talk, we will discuss some recent progress on the oriented diameter of some graph classes including connected graphs with minimum degree δ and planar triangulations.

The Existence of a Spanning Jellyfish

Ruth Luo

University of South Carolina

A jellyfish is a cycle C and a set of vertices X outside of C that have a common neighbor in C. In this talk, we present sufficient degree conditions for a graph to have a spanning jellyfish. This is joint work with A. Kostochka and J. Kim.

Refined Absorption: Further Applications

Luke Postle

University of Waterloo

We discuss further applications of the recently introduced method of "refined absorption" to combinatorial design theory. In particular, we discuss applications to high girth designs, thresholds for designs, and finding clique decompositions of random graphs and random regular graphs. Joint work with Michelle Delcourt and Tom Kelly.

Apexing and Edge-Apexing in Graphs

Jagdeep Singh

Binghamton University

A class of graphs is called hereditary if it is closed under taking induced subgraphs. Its apex class is defined as the class of graphs G that contain a vertex v such that $G - v$ is in the hereditary class. In recent work, Vaidy Sivaraman, Tom Zaslavsky, and I showed that if a hereditary class has finitely many forbidden induced subgraphs, then so does its apex class. I will talk about this result, its edge version, and a matroid analogue.

New Ramsey Multiplicity Bounds and Search Heuristics

Tibor Szabó

Freie Universität Berlin

We study two related problems concerning the number of monochromatic cliques in two-colorings of the complete graph that go back to questions of Erdős. Most notably, we "significantly" improve the best known upper bounds on the Ramsey multiplicity of K_4 and K_5 and settle the minimum number of independent sets of size four in graphs with clique number at most four. Motivated by the elusiveness of the symmetric Ramsey multiplicity problem, we also introduce the off-diagonal variant and obtain tight results when counting monochromatic K_4 or K_5 in only one of the colors and triangles in the other. The extremal constructions turn out to be blow-ups of finite graphs and were found through search heuristics. They are complemented by lower bounds and stability results established using flag algebras, resulting in a fully computer-assisted approach.

More broadly, these problems lead us to the study of the region of possible pairs of clique and independent set densities that can be realized as the limit of some sequence of graphs. Joint work with Olaf Parczyk, Sebastian Pokutta, and Christoph Spiegel.

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